

The dark energy equation of state

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ABSTRACT

We perform a study of cosmic evolution with an equation of state parameter $\omega(t) = \omega_0 + \omega_1(t\dot{H}/H)$ by selecting a phenomenological Λ model of the form, $\dot{\Lambda} \sim H^3$. This simple proposition explains both linearly expanding and inflationary Universes with a single set of equations. We notice that the inflation leads to a scaling in the equation of state parameter, $\omega(t)$, and hence in equation of state. In this approach, one of its two parameters have been pin pointed and the other have been delineated. It has been possible to show a connection between dark energy and Higgs-Boson.

Key words: gravitation - cosmological parameters - cosmology: theory - early Universe.

1 INTRODUCTION

Cosmological research is mainly concerned with time (and in some cases space as well) evolution of various physical parameters like scale factor, Hubble parameter, matter-energy density etc. Along with these parameters, in recent years a new physical entity Λ has resurrected in the foreground of cosmology. In fact, Λ has become an essential part of the field equations of Einstein after some observational results (Riess et al. 1998; Perlmutter et al. 1999) indicated towards an accelerating Universe. It is believed by most of the physicists that the cosmological parameter Λ is responsible for driving the present acceleration because it can exert negative pressure. Moreover, due to some fine-tuning problem (known as cosmological constant problem), Λ is regarded as a variable quantity rather than a constant.

Now, in order to specify exact time-dependence of the unknown physical quantities including Λ , one has to take recourse of a relationship between cosmic pressure p and matter-energy density ρ involving the equation of state parameter ω . Mathematically speaking, one variable quantity can depend on the product of two other variable quantities. So, one may construct ω as a function of time, red-shift or scale factor (Chervon & Zhuravlev 2000; Zhuravlev 2001; Peebles & Ratra 2003). In fact, values of ω at different stages of cosmic evolution suggest that it may evolve with time. As an instance, for the present pressure-less Universe, the value of ω is considered as zero, whereas its value was $1/3$ in the early radiation dominated Universe. However, it is convenient to consider ω as a constant quantity because observational data can hardly distinguish between a varying and a constant equation of

state (Kujat et al. 2002; Bartelmann et al. 2005). Here some useful limits on ω as appeared from SNIa data are $-1.67 < \omega < -0.62$ (Knop et al. 2003) whereas refined values were indicated by the combined SNIa data (with CMB anisotropy) and galaxy clustering statistics which is $-1.33 < \omega < -0.79$ (Tegmark et al. 2004).

As stated above, ω may have a functional relationship with scale factor or cosmological redshift. In connection to redshift it may depend linearly, $\omega(z) = \omega_o + \omega'z$, where $\omega' = (d\omega/dz)_{z=0}$ (Huterer & Turner 2001; Weller & Albrecht 2002) or it may have a non-linear relationship as $\omega(z) = \omega_o + \omega_1 z/(1+z)$ (Polarski & Chevallier 2001; Linder 2003). This suggests for a simple form

$$\omega(t) = \omega_0 + \omega_1(t\dot{H}/H), \quad (1)$$

which has got an explicit time dependence that disappears with the condition, $t\dot{H} = H$.

Using above proposition, we explore the physical features of different stages of cosmic evolution, viz., linearly expanding and inflationary Universes. For this, a phenomenological Λ model is selected to solve the Einstein field equations. There are mathematically motivated works (Ray, Mukhopadhyay & Duttachowdhury 2007; Mukhopadhyay, Ray & Duttachowdhury 2005, 2007; Mukhopadhyay, Ghosh, Khlopov & Ray 2007), wherein several phenomenological Λ models have been investigated for time-dependent ω .

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2 FIELD EQUATIONS FOR A STATIC SPHERICALLY SYMMETRIC SOURCE

The Einstein field equations are

$$R^{ij} - \frac{1}{2}Rg^{ij} = -8\pi G \left[T^{ij} - \frac{\Lambda}{8\pi G} g^{ij} \right], \quad (2)$$

where Λ is the time-dependent cosmological term with vacuum velocity of light being unity in relativistic units.

From equation (2) and Robertson-Walker metric, we get the Friedmann and Raychaudhuri equations, respectively

$$3H^2 + \frac{3k}{a^2} = 8\pi G\rho + \Lambda, \quad (3)$$

$$3H^2 + 3\dot{H} = -4\pi G(\rho + 3p) + \Lambda. \quad (4)$$

Here, $a = a(t)$ is the scale factor and k is the curvature constant which assumes values $-1, 0$ and $+1$ for open, flat and closed models of the Universe respectively. Also, $H = \dot{a}/a$ is the Hubble parameter and G, ρ, p are the gravitational constant, matter energy density and pressure respectively. However, the generalized energy conservation law for variable G and Λ is derived by Shapiro, Solà & Štefančič (2005) using Renormalization Group Theory and also by Vereschagin & Yegorian (2006) using a formula of Gurzadyan & Xue (2003). For variable Λ and constant G , the generalized conservation law reduces to the form

$$\dot{\rho} + 3(p + \rho)H = -\dot{\Lambda}/(8\pi G). \quad (5)$$

3 COSMOLOGICAL MODELS FOR VARIABLE EQUATION OF STATE PARAMETER

The barotropic equation of state which relates the pressure and density of the physical system is given by

$$p = \omega\rho. \quad (6)$$

Using this equation with equation (5), we arrive at

$$8\pi G\dot{\rho} + \dot{\Lambda} = -24\pi G(1 + \omega)\rho H. \quad (7)$$

For a flat Universe ($k = 0$), equation (3) yields

$$-4\pi G\rho = \dot{H}/(1 + \omega). \quad (8)$$

The equivalence of three phenomenological Λ -models (viz., $\Lambda \sim (\dot{a}/a)^2$, $\Lambda \sim \ddot{a}/a$ and $\Lambda \sim \rho$) have been studied in detail by Ray, Mukhopadhyay & Duttachowdhury (2007) for constant ω . So, it is reasonable to study a variable- Λ model with a variable ω . Let us, therefore, use the *ansatz* $\dot{\Lambda} \propto H^3$, so that

$$\dot{\Lambda} = AH^3. \quad (9)$$

This *ansatz* may find realization in the framework of self consistent inflation model (Dymnikova & Khlopov 2000, 2001), in which time-variation of Λ is determined by the rate of Bose condensate evaporation (Dymnikova & Khlopov 2000) with $A \sim (m_B/m_P)^2$ (where m_B is the mass of bosons and m_P is the Planck mass).

From equations (4),(6),(8) and (9), we get

$$\frac{2}{(1 + \omega)H^3} \frac{d^2H}{dt^2} + \frac{6}{H^2} \frac{dH}{dt} = A. \quad (10)$$

With $dH/dt = \dot{H}$, equation (10) reduces to

$$\frac{d\dot{H}}{dH} + 3(1 + \omega)H = \frac{A(1 + \omega)H^3}{2\dot{H}}. \quad (11)$$

We would now show, how does these field equations used in

conjunction with our proposition (equation 1) incorporate both linearly expanding and inflationary Universes.

4 LINEARLY EXPANDING UNIVERSE

We consider a situation in which our Universe started expanding linearly (Crane 1979; Azuma & Tomimatsu 1982; Calzetta & Castagnino 1983) since its very beginning at a rate $\dot{H} = dH/dt$ with $H(t = 0) = 0$ at the point of singularity. Thus at a later time $t > 0$, the observable $H(t)$ would be determined by the relation, $H(t) = t\dot{H}$. The \dot{H} is the present value of H divided by the age of the Universe. In this case, equation (11) reduces to

$$\frac{d\dot{H}}{dH} + 3(1 + W)H = \frac{A(1 + W)H^3}{2\dot{H}} \quad (12)$$

where $W = \omega_0 + \omega_1$.

Solution set for the differential equation (12) in connection to different physical parameters is given below,

$$a(t) = C(Et + D)^{1/E}, \quad (13)$$

$$H(t) = \frac{1}{Et + D}, \quad (14)$$

$$\omega(t) = \omega_0 + \omega_1 \left(\frac{1}{1 + \frac{D}{Et}} \right), \quad (15)$$

$$\rho(t) = \frac{E}{4\pi G(Et + D)^2(1 + \omega(t))}, \quad (16)$$

$$p(t) = \omega(t)\rho(t), \quad (17)$$

$$\Lambda(t) = -\frac{A}{2E(Et + D)^2}. \quad (18)$$

Here, C and D are integration constants and E reads as

$$E = \left[3(1 + W) + \sqrt{9(1 + W)^2 + 4A(1 + W)} \right] / 4. \quad (19)$$

With the fact that $A \ll W$, we may neglect the term involving A in the above equation, which would yield $E \approx 3(1 + W)/2$. However, this would amount to be neglecting r.h.s term, $A(1 + W)H^3/2\dot{H}$, of equation (12), which suggests that the effect of this term is small. It is also obvious from equation (12) that this term matters only at an early stage of the evolution of the Universe where $H \sim A$. However, at this regime quantum effects become important and hence are of no relevance in our general relativistic approach.

With the consideration, $H(t) = \dot{H}$, equation (1) does not involve any explicit time dependence. So is equation (15) provided $D = 0$. We notice that with $E = 1$ and integration constants $D = 0$ and $C = 1$, equation (13) becomes a perfect example of a linearly expanding Robertson-Walker Universe, $a(t) = t$. However, $E = 1$ suggests a value $W = w_0 + w_1 = \omega(t) = -1/3$, which is well above the minimum limit of $\omega(t)$ i.e. -0.79 . We would see it later that inflation scales it to a lower value. From equation (14), deceleration parameter, q , is deduced to be $q = E - 1$, which thus is zero for such a linearly expanding Universe.

5 INFLATIONARY UNIVERSE

We now consider a physical situation in which our Universe initially inflated non-linearly up to a certain value of time $t = t_0 \ll 1$ second (Guth 1981; Linde 1982; Albrecht & Steinhardt 1982). Since this time onward the expansion of the Universe is assumed to be quite linear, which is described by the rate $\dot{H} = dH/d\tau$. Here

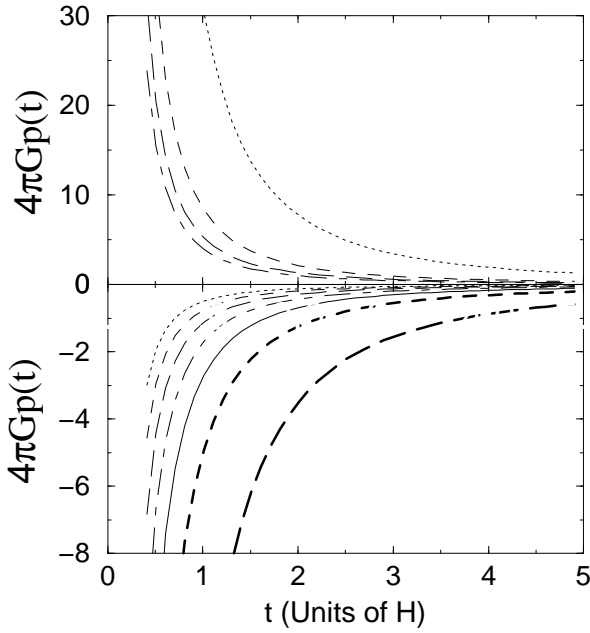


Figure 1. The upper panel represents $1 + \omega(t) < 0$. In this panel, dotted, dashed, long-dashed, and chain curves correspond to $\omega_1 = -0.7, -0.8, -0.9$ and 1.0 , respectively. In the lower panel representing $1 + \omega(t) > 0$, same curves correspond to $\omega_1 = 0.0, -0.1, -0.2$ and -0.3 , respectively. The solid, thick dashed and thick long-dashed lines represent $\omega_1 = -0.4, -0.5$ and -0.6 , respectively. For all these ω_0 is taken to be $-1/3$.

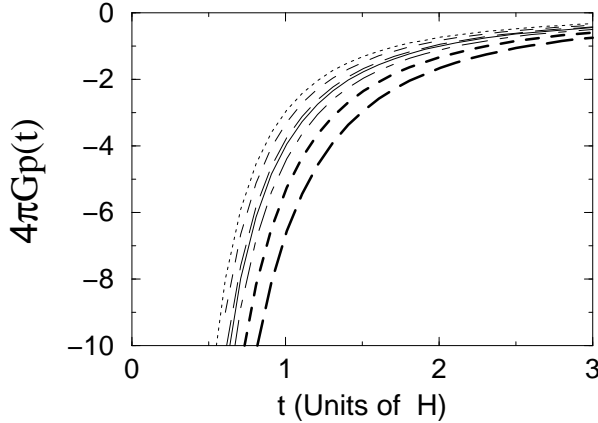


Figure 2. The dotted, dashed, long-dashed, chain and solid curves represent $\omega_0 = -0.1, -0.2, -0.3, -0.4$ and $-1/3$, respectively. The thick dashed and thick long-dashed lines represent $\omega = -0.5$ and -0.6 , respectively. For all these, ω_1 is adjusted using $\omega(t) = \omega_0 + \omega_1 = -0.8$

τ is the measure of the time from $t = t_0$. This leads to a translation in H such that $H(t = t_0 + \tau) = H(t_0) + \tau \dot{H}$. We assume that inflation has led to a condition $H(t_0) \gg \tau \dot{H}$, which implies that $H(t) = H(t_0 + \tau) \gg \tau \dot{H}$. With the consideration that the period of inflation has been very very brief compared to the age of the Universe, we may write $t \approx t_0 + \tau$ and $\dot{H} = dH/dt \approx dH/d\tau$. However, the value of \dot{H} would be different from the previous case of linearly expanding Universe. Under these conditions, equation (11) reduces to

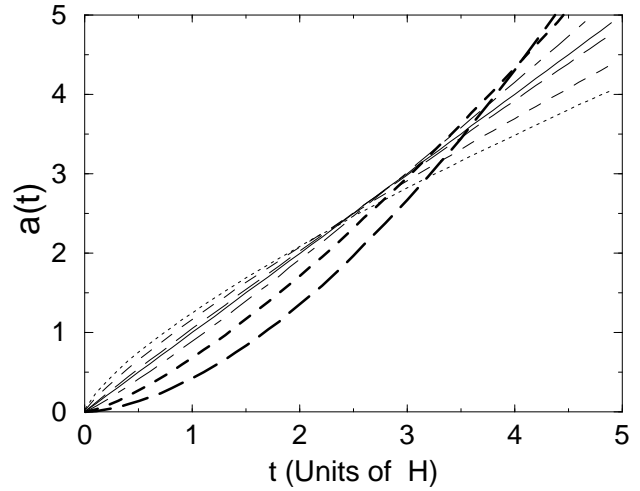


Figure 3. The scale factors for the curves shown in Figure 2.

$$\frac{d\dot{H}}{dH} + 3(1 + \omega_0)H = \frac{A(1 + \omega_0)H^3}{2\dot{H}}. \quad (20)$$

If we substitute W at the place of ω_0 in equation (12), we arrive at equation (20). The solution set obtained for the linearly expanding Universe is still valid for the inflationary Universe provided we substitute ω_0 at the place of W in equation (19). This scaling from W to ω_0 in equation (19) may be attributed to the adiabatic expansion of the Universe till time t_0 . The r.h.s. of equation (20) may be always neglected in this case because H is evolved to a large value compared to the values of A during inflation.

With the consideration that $A \ll \omega_0$, we obtain $\omega_0 = -1/3$. Thus, the value $\omega(t) = \omega_0 + \omega_1 = -1/3$ as obtained for linearly expanding Universe now corresponds to $\omega_0 = \omega(t) - \omega_1 = -1/3$ for an inflationary Universe. Therefore, the values $\omega_0 = -1/3$ and $\omega_1 = 0$ correspond to previously discussed linearly expanding Universe and a nonzero value for ω_1 represents inflationary Universe. Thus, we notice a direct correlation between $\omega(t)$ and the inflation of the Robertson-walker Universe, which is buried in the value of the parameter ω_1 . With $\omega_0 = -1/3$, the range of the values $-1.0 < \omega_1 < -0.46$ falls in the suggested range $-1.33 < \omega(t) < -0.79$.

We may invoke a time dependence in equation (15) through D . However, as mentioned earlier, data do not suggest any significant explicit time dependence in $\omega(t)$, thus D is set to zero. The non-linearity in $a(t)$ may be invoked through ω_0 in E by choosing a different value for it other than $-1/3$. Thus for a linear behaviour after inflation this value is fixed to $-1/3$. The equation (16) for ρ is singular at $1 + \omega(t) = 0$. So is equation (17) for p , which has been plotted in Figure 1. For the negative pressure, as required by the dark energy, it applies a constraint on $\omega(t)$ such that $\omega(t) > -1$ or $\omega_1 = \omega(t) - \omega_0 > -2/3$. We find a range $-2/3 < \omega_1 < -0.46$ with $\omega_0 = -1/3$.

6 DISCUSSION AND REMARKS

We have discussed two Universes: (i) a linearly expanding Universe from its very beginning, (ii) and also the Universe like ours, which has gone through an inflation at its very early stage followed by a linear expansion later. We notice that these two kind of Universes, which are direct consequence of our proposition (equation 1), are

represented by the same set of equations with a translational shift in the equation of state parameter in the latter case compared to the former. In both the cases, $a(t) = 1$ demands $E = 1$, which applies a constraint on the equation of state parameter. For the inflationary Universe, we have pin pointed $\omega_0 = -1/3$ and have delineated the other parameter with a range $-2/3 < \omega_1 < -0.46$. We observe that former is a special case of the latter with $\omega_0 = -1/3$ and $\omega_1=0$. Any other value of ω_0 would invoke a non-linear behaviour in $a(t)$ through E . The effect of the variation of ω_0 on p is presented in Figure 2 for a constant $\omega = \omega_0 + \omega_1 = -0.80$ obtained by adjusting ω_1 accordingly. The ω_1 has nothing to do with E and hence has nothing to do with $a(t)$. However, its value is a measure of translation in ω due to inflation. The equations for ρ and p involve ω and hence would remain unchanged with its constant value. Thus, variations in curves of Figure 2 is purely due to the variation in ω_0 . The corresponding variations in $a(t)$ are shown in Figure 3.

A negligible value of A is shown to be physically possible from the viewpoint of cosmology and particle physics, which means the absence of Λ in the field equations. So, both from physical and mathematical point of view the nullity of Λ is achieved for the same Λ model. Again, the expression of q in this case has a striking similarity with that of Ray, Mukhopadhyay & Duttachowdhury (2007). This work suggests that in the late phase of the Universe, where $t\dot{H} = H$, the equation of state parameter behaves as a constant. Perhaps for this reason current data cannot distinguish clearly between a time-dependent ω and a constant one as pointed out by some workers (Kujat et al. 2002; Bartelmann et al. 2005).

Separating the entire cosmic history into two phases, it has been possible to derive the time-dependent expressions for the scale factor and the other physical parameters of each phase. It has been found that for inflationary phase, the deceleration parameter q depends on time whereas for the linearly expanding phase it is constant, rather zero. This supports the opinion that q has changed during the course of time (Riess et al. 2001; Amendola 2003; Padmanabhan & Roychowdhury 2003).

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